Ginzburg-Landau theory

A phenomenological theory of the superconductive transition. It is the 2nd order phase transition.

There must be some order parameter of this transition. It will be the wavefunction $\Psi(r)$ of the „super" electrons.

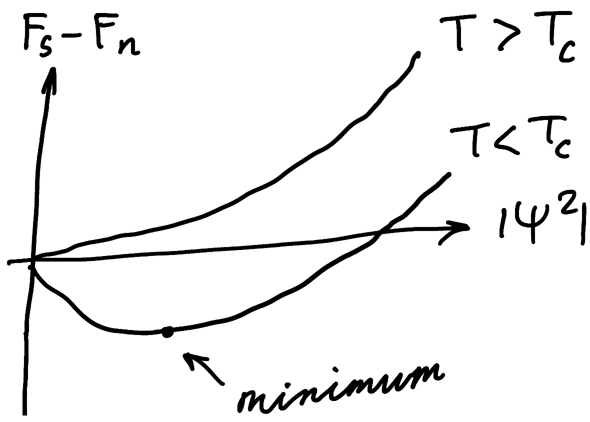
$\exists\ \Psi(\vec{r})$ - the w.f. of Cooper pairs

$$|\Psi(\vec{r})|^2 = \frac{n_s}{2} \quad \text{(in a homogeneous system)}$$

Near T_c (zero magnetic field)

$$F_s = F_n + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4$$

The coefficient α must change sign at $T = T_c$ in order for the order parameter $|\Psi|^2$ to have a finite order parameter below T_c and zero order parameter above T_c



The condition of minimum : $\dfrac{dF}{d|\Psi|^2} = 0$

$$\alpha + \beta|\Psi|^2 = 0 \rightarrow |\Psi^2| = -\frac{\alpha}{\beta}$$

So, $\alpha = \mathcal{I}(T - T_c)$

$$\boxed{F_s = F_n + \mathcal{I}(T - T_c)|\Psi|^2 + \beta|\Psi|^4 + \ldots}$$

More generically, the density of the free energy is given by

$$F_{sH} = F_n + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \underbrace{\frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} - \frac{2e}{c}\vec{A}\right)\Psi\right|^2}_{\text{Kinetic energy}}$$
$$+ \frac{H^2}{8\pi} - \frac{\vec{H}\vec{H}_0}{4\pi}$$

Let's assume the magnetic field is zero (and $\vec{A} = 0$)

Then we have to find the variational derivative of $\int \left[\alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}|(-i\hbar\vec{\nabla})\Psi|^2\right] dV$

$$\int dV \left[\alpha\Psi + \beta\Psi|\Psi|^2 + \frac{1}{2m^*}(-i\hbar\nabla)^2\Psi\right]\delta\Psi^* = 0$$

Introduce $\xi^2 = \dfrac{\hbar^2}{2m^*|\alpha|}$

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$$\xi^2 (i\nabla)^2 \Psi - \Psi + \Psi|\Psi|^2 = 0$$

Recovering the vector potential $\vec{A}$

$$\boxed{\xi^2 \left(i\vec{\nabla} + \frac{2\pi}{\varphi_0}\vec{A}\right)^2 \Psi - \Psi + \Psi|\Psi|^2 = 0}$$

Ginzburg-Landau equation

where $\varphi_0 = \dfrac{\pi\hbar c}{e}$

ξ - the coherence length, $\propto (T_c - T)^{-\frac{1}{2}}$

Proximity effect

The GL equation for x varying along one direction only

$$-\xi^2 \frac{d^2\Psi}{dx^2} - \Psi + \Psi^3 = 0$$

Easy to integrate: $-\xi^2 \left(\frac{d\Psi}{dx}\right)^2 - \Psi^2 + \frac{1}{2}\Psi^4 = C$

At $x \to +\infty$

$\dfrac{d\Psi}{dx} = 0$, $\Psi \to 1$

and $C \to -\frac{1}{2}$

$$\Psi = \tanh \frac{x - x_0}{\sqrt{2}\,\xi}$$

Inside the normal region $\psi \ll 1$

$$-\xi_n^2 \left(\frac{d\psi}{dx}\right)^2 + \psi = 0 \rightarrow \psi = \psi_0 e^{-\frac{|x|}{\xi_n}}$$